Binomial Expansion- Questions

May 2016 Mathematics Advanced Paper 1: Pure Mathematics 2

1.

5. (*a*) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 9x)^4,$$

giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \quad \text{where } k \text{ is a constant.}$$

The expansion, in ascending powers of x, of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2,$$

where A and B are constants.

(*b*) Write down the value of A.

(1)

(*c*) Find the value of k.

(2)

(*d*) Hence find the value of B.

(2)

May 2014 Mathematics Advanced Paper 1: Pure Mathematics 2

2.

3. (*a*) Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(2 - 3x)^6$, giving each term in its simplest form.

(4)

(*b*) Hence, or otherwise, find the first 3 terms, in ascending powers of x, of the expansion of

$$\left(1 + \frac{x}{2}\right)(2 - 3x)^6.$$

(3)

May 2013 Mathematics Advanced Paper 1: Pure Mathematics 2

3.

2. (*a*) Use the binomial theorem to find all the terms of the expansion of

$$(2 + 3x)^4.$$

Give each term in its simplest form.

(4)

(*b*) Write down the expansion of

$$(2 - 3x)^4$$

in ascending powers of x, giving each term in its simplest form.

(1)

Jan 2012 Mathematics Advanced Paper 1: Pure Mathematics 2

4.

3. (*a*) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^8,$$

giving each term in its simplest form.

(4)

(*b*) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

May 2011 Mathematics Advanced Paper 1: Pure Mathematics 2

5.

2. (*a*) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 + bx)^5$$

where b is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^2 is twice the coefficient of x,

(*b*) find the value of b.

(2)

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 2

6.

5. Given that $\binom{40}{4} = \frac{40!}{4!b!}$,

(a) write down the value of b.

(1)

In the binomial expansion of $(1 + x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\frac{q}{p}$.

(3)

Jun 2010 Mathematics Advanced Paper 1: Pure Mathematics 2

7.

4. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1+ ax)^7$, where a is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 in this expansion is 525,

(b) find the possible values of a.

(2)

June 2017 Mathematics Advanced Paper 1: Pure Mathematics 4

8.

2. $f(x) = (2 + kx)^{-3}$, $|kx| < 2$, where k is a positive constant

The binomial expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2 is

$$A + Bx + \frac{243}{16}x^2$$

where A and B are constants.

(a) Write down the value of A.

(1)

(b) Find the value of k.

(3)

(c) Find the value of B.

(2)

June 2016 Mathematics Advanced Paper 1: Pure Mathematics 4

9.

1. Use the binomial series to find the expansion of

$$\frac{1}{(2+5x)^3}, \quad |x| < \frac{2}{5},$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a fraction in its simplest form.

(6)

June 2015 Mathematics Advanced Paper 1: Pure Mathematics 4

10.

1. (*a*) Find the binomial expansion of

$$(4+5x)^{\frac{1}{2}}, \quad |x| < \frac{4}{5},$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient in its simplest form.

(5)

(*b*) Find the exact value of $(4+5x)^{\frac{1}{2}}$ when $x = \frac{1}{10}$.

Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

(1)

(*c*) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (*a*) and hence find an approximate value for $\sqrt{2}$.

Give your answer in the form $\frac{p}{q}$, where p and q are integers.

(2)

June 2014 Mathematics Advanced Paper 1: Pure Mathematics 4

11.

2. Given that the binomial expansion of $(1 + kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \ldots$$

(a) find the value of the constant k,

(2)

(b) find the value of the constant A, giving your answer in its simplest form.

(3)

June 2013 Mathematics Advanced Paper 1: Pure Mathematics 4

12.

2. (a) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1+x+\frac{1}{2}x^2, \qquad |x| < 1$$

(6)

(b) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1+x+\frac{1}{2}x^2$$

to obtain an approximation to $\sqrt{3}$.

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(3)

Jan 2013 Mathematics Advanced Paper 1: Pure Mathematics 4

13.

1. Given

$$f(x) = (2 + 3x)^{-3}, \quad |x| < \frac{2}{3},$$

find the binomial expansion of $f(x)$, in ascending powers of x, up to and including the term in x^3.

Give each coefficient as a simplified fraction.

(5)

June 2012 Mathematics Advanced Paper 1: Pure Mathematics 4

14.

3. $$f(x) = \frac{6}{\sqrt{(9-4x)}}, \qquad |x| < \frac{9}{4}.$$

(*a*) Find the binomial expansion of f(x) in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(6)

Use your answer to part (*a*) to find the binomial expansion in ascending powers of x, up to and including the term in x^3, of

(*b*) $g(x) = \frac{6}{\sqrt{(9+4x)}}, \qquad |x| < \frac{9}{4},$

(1)

(*c*) $h(x) = \frac{6}{\sqrt{(9-8x)}}, \qquad |x| < \frac{9}{8}.$

(2)

Jan 2012 Mathematics Advanced Paper 1: Pure Mathematics 4

15.

3. (*a*) Expand

$$\frac{1}{(2-5x)^2}, \qquad |x| < \frac{2}{5},$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

(5)

Given that the binomial expansion of $\frac{2+kx}{(2-5x)^2}$, $|x| < \frac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \ldots,$$

(*b*) find the value of the constant k,

(2)

(*c*) find the value of the constant A.

(2)

June 2011 Mathematics Advanced Paper 1: Pure Mathematics 4

16.

2. $$f(x) = \frac{1}{\sqrt{(9+4x^2)}}, \qquad |x| < \frac{3}{2}.$$

Find the first three non-zero terms of the binomial expansion of f(x) in ascending powers of x. Give each coefficient as a simplified fraction.

(6)

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 4

17.

5. (a) Use the binomial theorem to expand

$$(2-3x)^{-2}, \qquad |x| < \frac{2}{3},$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(5)

$$f(x) = \frac{a+bx}{(2-3x)^2}, \quad |x| < \frac{2}{3}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of f(x), in ascending powers of x, the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$.

Find

(b) the value of a and the value of b,

(5)

(c) the coefficient of x^3, giving your answer as a simplified fraction.

(3)

Jan 2010 Mathematics Advanced Paper 1: Pure Mathematics 4

18.

1. (*a*) Find the binomial expansion of

$$\sqrt{(1-8x)}, \quad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3, simplifying each term.

(4)

(*b*) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1-8x)}$ is $\frac{\sqrt{23}}{5}$.

(2)

(*c*) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (*a*) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places.

(3)